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EROSION OF THE DEFENSE INDUSTRIAL BASE: AN ECONOMETRIC STUDY OF THE FERROUS FOUNDRY INDUSTRY

Harold J. Brumm, Jr.

October 1977

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This study is an empirical investigation of the economic effect of regulations imposed by EPA and OSHA on the ferrous foundry industry. Its inception was motivated by expressed concern of some officials in DIRSO and OASD/MRA&L about the impact of federal environmental, health, and safety legislation on firms comprising the defense industrial base. The econometric methodology employed consists of statistical estimation of the parameters of various supply-and-demand models of the

20. (continued)

market for iron and steel castings.

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FOREWORD

This paper is an empirical examination of the economic effect of EPA and OSHA regulations on the ferrous foundry industry. Its inception was motivated by expressed concern of some officials (namely in DIRSO--the Defense Industrial Resources Support Office, and OASD/MRA&L--the Office of the Assistant Secretary of Defense for Manpower, Reserve Affairs and Logistics) about the impact of federal environmental, health, and safety legislation on firms comprising the defense industrial base. The foundries in particular allegedly have been hard hit by federal environmental regulations and safety and health directives.

The econometric methodology employed in this report was derived from the emerging literature on statistical estimation of models of markets in disequilibrium. The principal conclusion is that EPA and OSHA regulations have had a negative impact on the foundries' operations. For reasons presented herein, however, this conclusion is tentative. More data and better quality data will have to be analyzed before any strict conclusions can be drawn.

This paper has benefited greatly from the comments of R. William Gilmer and William J. Raduchel. The author bears sole responsibility for the analyses, views, and conclusions presented. Nothing contained herein necessarily represents the official position of IDA or any of its Department of Defense sponsors.

EROSION OF THE DEFENSE INDUSTRIAL BASE: AN ECONOMETRIC STUDY OF THE FERROUS FOUNDRY INDUSTRY

Recently, Department of Defense (DoD) policy-makers have expressed concern that this nation's defense industrial base is shrinking at an alarming rate.[1] It has been asserted that this erosion process is particularly pronounced in the ferrous foundry "industry,"[2] resulting in a shortage of certain defense-specific products, e.g., steel castings for tank turrets. Since a foundry is one of the least safe workplaces in the United States,[3] and since a foundry that uses older technologies (e.g., a cupola furnace rather than the newer electric induction furnace) is an air polluter *par excellence*,[4] one could anticipate that regulations imposed recently by the Occupational Safety and Health Administration (OSHA) and by the Environmental Protection Agency (EPA) would impact significantly on this type of firm. Indeed, a reasonable hypothesis is that OSHA and EPA

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[1]See, e.g., Jacques S. Gansler, Deputy Assistant Secretary of Defense (Material Acquisition), *White Paper on the Defense Industrial Base*, June 8, 1976, pp. 4-5, 9. (Mimeographed.)

[2]The notion of a ferrous foundry "industry" is, of course, a fiction because the outputs—castings, molds, pipes, and fittings—of the various types of foundries are heterogeneous. However, for the purposes of this study, the gray iron, malleable iron, and steel foundries (SIC Codes 3321, 3322, and 3323, respectively) will be lumped together in one "industry."

[3]U.S. Department of Labor, Bureau of Labor Statistics, *Occupational Injuries and Illnesses by Industry, 1973*, Bulletin 1830, March 1974.

[4]Emissions from an uncontrolled cupola have been reported to exceed 17.4 pounds of particles per ton of melt. See U.S. Department of Health, Education, and Welfare, Public Health Service, Environmental Health Service, National Air Pollution Control Administration, *Economic Impact of Air Pollution Controls on Gray Iron Foundry Industry* (Washington, D.C.: GPO, 1970), p. 7.

regulations have increased the typical foundry's total cost of production and, as a result, have reduced the defense-related output of the ferrous foundry industry.

Shortly after this research was begun it was discovered that an attempt to empirically test this hypothesis would be severely constrained by the paucity of relevant data. As a result, drastic simplifications in the analysis were required. The *deus ex machina* adopted for this analysis is a simple demand-and-supply model. The model's theoretical foundations are outlined in Section A, while econometric considerations that are germane to this study are presented in Section B. Finally, Section C presents this study's empirical findings and conclusions.

A. FIRM AND INDUSTRY RESPONSE TO EPA AND OSHA REGULATION

There exist two fairly distinct groups of firms comprising the ferrous foundry industry--jobbing foundries and production foundries.[1] Job shop foundries tend to be relatively small operations, usually family-owned or held as small closed corporations without outside equity capital, and producing a wide variety of castings that vary by quantity, size, weight, and technical specifications. This lack of specialization in terms of output greatly reduces the possibilities for mechanization of the job shop's production techniques. As a result, 40 to 70 man-hours of labor input may be expended per ton of castings shipped. By contrast, production foundries, which also produce a wide variety of castings, tend to be less labor intensive and use extensively mechanized production techniques. The labor input expended per ton of castings shipped usually is in the range of 15 to 30 man-hours.

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[1]A third group of foundries could be broken out from these two, *viz.*, cast iron pipe foundries. Of course, even more detailed taxonomies of the "industry" are possible.

In order to examine at a theoretical level the impact of EPA and OSHA regulations on the "typical" foundry, it will be convenient to make several simplifying assumptions at the outset. First, let us assume that the market for iron and steel castings--the principal output of the ferrous foundries--is perfectly competitive.[1] This assumption would appear to be a reasonable first approximation of the industry's market structure. On the *supply* side, the industry includes a large number of many small firms. At the present time, 82 percent of all foundries employ less than 100 workers; 50 percent employ fewer than 20 workers.[2] On the *demand* side, iron and steel castings are commonly intermediate to a wide variety of final manufactured goods. For example, metal castings are required as end products or component parts of 90 percent of all durable goods manufactured in the United States.[3] Furthermore, most of the industry's output is purchased by private firms. For example, less than five percent of that output was purchased by local, state, and federal government agencies in 1972.[4]

An effect of EPA and OSHA regulations that remains unaltered for a specified interval of time is to require the foundry to purchase pollution abatement and safety equipment. Thus, for a given time period, this expenditure requirement is tantamount to a lump-sum tax, i.e., a tax which does not vary with the firm's level of production or profit. Such a tax appears as a constant subtracted from the firm's total revenue for the given

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[1]The products of this "industry" are not homogeneous. Strictly speaking, therefore, at least one of the assumptions of the model of perfect competition is violated.

[2]Debbie Tennison, "The Foundry Industry--Achilles' Heel of Defense?" *National Defense,* Vol. 60 (March-April 1976), pp. 366-69.

[3]*Ibid.*

[4]U.S. Department of Commerce, Bureau of the Census, *1972 Census of Manufactures, Industry Series: Ferrous and Nonferrous Foundries--SIC Industry Groups 332 and 336,* MC72(2)-33B (Washington, D.C.: GPO, 1974), pp. 33B3-33B4.

period. Other things being equal, as long as the level of the tax does not change, the firm's level of production will remain unaltered--provided that the tax does not raise the foundry's overall costs to such a level that the firm is forced out of business. A lump-sum tax that remains constant during a given period does not affect any surviving firm's production level because the production level that maximized net revenue before the subtraction of a constant also maximizes net revenue after subtraction.[1] Such a tax cannot affect the firm's internal allocation of its resources; it can only influence whether to operate or shut down.

Suppose, however, that with the passage of time the level of the lump-sum tax is increased. This assumption seems reasonably consistent with the casual observation that EPA and OSHA regulations imposed on foundries have become increasingly stringent during the past few years. As the required expenditures on pollution abatement and safety equipment rise, the firm's long-run average cost will increase for every level of output. Hence, the long-run supply price for the industry will increase and industry output will decline, other things being equal. This reduction in aggregate output will be accomplished by an exodus of firms from the industry. The traditional economic theory of the firm predicts exactly that result.[2]

B. ECONOMETRIC CONSIDERATIONS

Our present research certainly is not without its limitations. The scarcity of data not only hinders model formulation but also poses unknown dangers associated with errors in the equations that are statistically estimated here. Data limitations drastically constrain the specification of any structural model of the market for castings.

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[1] We are implicitly assuming that the firm's objective is to maximize profit, i.e., net revenue.

[2] The theoretical conclusions of this section are derived in Appendix B.

According to received theory and given the state of technology the determinants of the market supply of a commodity are its market price, the market prices of the resources used to produce it, and the number of firms engaged in its production.[1] Given the critics' allegations, the deleterious impact of EPA and OSHA regulations on the foundries would result in a reduction in the number of those firms. Thus, a variable that captures that alleged impact should be included as an argument of the supply function. Of course, in all likelihood not all firms in the industry would feel the impact of those regulations simultaneously. However, according to an industry representative, EPA's regulations began impacting most foundries in May 1971, and OSHA's in March 1974.[2] Given the nature of EPA and OSHA regulations, their effect on the industry as a whole should be viewed not as a once and for all lump-sum tax but as a series of successive lump-sum taxes imposed over discrete intervals of time.

Before continuing further, it will be convenient to define the following variables:[3]

DEMAND = aggregate quantity demanded of iron and steel castings

SUPPLY = aggregate quantity supplied of iron and steel castings

QUAN = observed quantity of iron and steel castings

PRICE1 = price of iron and steel castings

PRICE2 = price of a composite good produced by purchasers of castings

WAGE1 = wage rate paid to iron and steel foundry workers

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[1]The derivation of the market supply function for castings is presented in Appendix B.

[2]Letter from Walter M. Kiplinger, Jr., Washington Representative, Cast Metals Federation, May 12, 1977.

[3]A detailed explanation of how these variables are actually measured is given in Appendix A.

WAGE2 = wage rate paid to workers employed by buyers of iron and steel castings

COST = price of nonlabor inputs

TREND = a time trend variable

REGDUM = a regulation-effects qualitative variable

REG = expenditures on pollution abatement and safety equipment

WPI = a proxy for the general price level.[1]

Two different linear specifications of the market supply function for castings are considered. In the first, the explanatory predetermined variables are z_2 = REGDUM, z_3 = WAGE1/WPI, z_4 = COST/WPI, and z_5 = TREND, while the explanatory endogenous variable is y_1 = PRICE1/WPI. The second specification employs the same explanatory variables except that z_2 is replaced by z_2^* = REG/WPI.

The market demand for castings can be obtained by aggregating the derived demand functions of the individual buyers of castings,[2] most of which are private industrial firms.[3] Strictly speaking, the model should make some provision for public as well as private purchases of the industry's output. However, since only a small percentage of the industry's output is purchased by government agencies each year, no attempt will be made herein to estimate (let alone theoretically derive!) a separate function for government demand for castings.

The quantity of castings demanded by each industrial purchaser depends upon the market price of that firm's product, the market price of castings, and the market prices of other

[1]The general price level is more extensive than is accounted for by current price indexes, which typically include prices of output but ignore the prices of current assets. See Armen A. Alchian and Benjamin Klein, "On a Correct Measure of Inflation," *Journal of Money, Credit and Banking,* Vol. 5 (Feb. 1973), pp. 173-91.

[2]The derivation of this demand function is presented in Appendix B.

[3]See *supra,* pp. 3-4.

inputs that the firm uses. In this study, the functional specification employed for aggregate demand for castings is a linear one that has the following explanatory predetermined variables: x_2 = WAGE2/WPI, x_3 = COST/WPI, x_4 = PRICE2/WPI, and x_5 = TREND. An additional argument of the supply function is the current endogenous variable y_1 = PRICE1/WPI.

For the various functional specifications employed, parameters were estimated by two methods: the conventional two-stage least squares (TSLS) technique[1] and Amemiya's two-stage least squares (ATSLS) scheme for estimating the parameters of a model of a market in disequilibrium.[2] With one exception (discussed below), the first method was used to estimate the coefficients in the first two models shown below and the second method was used to estimate the third and fourth models.

<u>Model 1</u>

$$\text{DEMAND}_t = \alpha_{10} + \alpha_{11} y_{1t} + \sum_{k=2}^{5} \alpha_{1k} x_{k,t-1} + \varepsilon_{1t}^{D} \tag{1}$$

$$\text{SUPPLY}_t = \beta_{10} + \beta_{11} y_{1t} + \sum_{j=2}^{5} \beta_{1j} z_{j,t-1} + \varepsilon_{1t}^{S} \tag{2}$$

$$\text{DEMAND}_t = \text{QUAN}_t = \text{SUPPLY}_t \tag{3}$$

<u>Model 2</u>

$$\text{DEMAND}_t = \alpha_{20} + \alpha_{21} y_{1t} + \sum_{k=2}^{5} \alpha_{k} x_{k,t-1} + \varepsilon_{2t}^{D} \tag{4}$$

$$\text{SUPPLY}_t = \beta_{20} + \beta_{21} y_{1t} + \beta_{22} z_{2t}^{*} + \sum_{j=3}^{5} \beta_{2j} z_{j,t-1} + \varepsilon_{2t}^{S} \tag{5}$$

$$\text{DEMAND}_t = \text{QUAN}_t = \text{SUPPLY}_t \tag{6}$$

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[1]See any econometrics textbook for a discussion of TSLS estimation.

[2]Takeshi Amemiya, "A Note on a Fair and Jaffe Model," *Econometrica*, Vol. 42 (July 1974), pp. 759-62.

Model 3

$$\mathrm{DEMAND}_t = \alpha_{30} + \sum_{k=2}^{5} \alpha_{3k} x_{k,t-1} + \varepsilon_{3t}^{D} \tag{7}$$

$$\mathrm{SUPPLY}_t = \beta_{30} + \sum_{j=2}^{5} \beta_{3j} z_{j,t-1} + \varepsilon_{3t}^{S} \tag{8}$$

$$\mathrm{QUAN}_t = \min(\mathrm{DEMAND}_t,\ \mathrm{SUPPLY}_t) \tag{9}$$

$$\Delta y_{1t} = \gamma_3(\mathrm{DEMAND}_t - \mathrm{SUPPLY}_t), \qquad \gamma_3 > 0 \tag{10}$$

Model 4

$$\mathrm{DEMAND}_t = \alpha_{40} + \sum_{k=2}^{5} \alpha_{4k} x_{k,t-1} + \varepsilon_{4t}^{D} \tag{11}$$

$$\mathrm{SUPPLY}_t = \beta_{40} + \beta_{42} z_{2t}^{*} + \sum_{j=3}^{5} \beta_{4j} z_{j,t-1} + \varepsilon_{4t}^{S} \tag{12}$$

$$\mathrm{QUAN}_t = \min(\mathrm{DEMAND}_t,\ \mathrm{SUPPLY}_t) \tag{13}$$

$$\Delta y_{1t} = \gamma_4(\mathrm{DEMAND}_t - \mathrm{SUPPLY}_t), \qquad \gamma_4 > 0 \tag{14}$$

The ε_t's are assumed to be spherical normally distributed random variables[1] that are serially and contemporaneously independent, the γ's are constants of proportionality whose values are unknown, and the subscript t refers to the t^{th} time period. Models 1 and 3 were estimated on the basis of monthly observations over the period January 1970 through October 1976, the last date for which data were available when this investigation began. This particular interval of time was chosen because, beginning with data for calendar year 1971, the Bureau of Labor Statistics (BLS) converted the various components of the Wholesale Price Index (WPI) from a 1957-59 to a 1967 reference base period. To obtain a few observations prior to May 1971, the

[1] A random variable is said to have the spherical normal distribution if it is a normal random variable distributed with mean zero and positive finite variance.

time when foundries evidently began to feel the pressures of EPA regulations, the WPI was multiplied times BLS's rebasing factor for each month in calendar year 1970.[1,2]

A good case can be made here for the use of a disequilibrium model. During the early 1970s, the period from which many of our data points were drawn, there existed considerable economic turbulence--recession, inflation, devaluation of the dollar, and so forth. Except for markets where the price mechanism was particularly efficient, shortages and surpluses developed. The possibility that during this period disequilibrium characterized the market for castings has important implications for the present study. As one noted economist has pointed out in another context, (while) "...it is of great convenience in fitting simultaneous equation[s] models to be able to assume that quantity supplied is equal to quantity demanded, ...where the world is not obliging enough to satisfy this condition, econometricians may be forced to go to the trouble of making more realistic assumptions."[3]

Although all variables in Models 1 and 3 are discussed in Appendix A, REGDUM deserves special attention here. It was set equal to zero for all months prior to May 1971, when EPA regulations supposedly began to seriously affect the foundries'

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[1] The values of the WPI rebasing factor are presented in U.S. Department of Labor, Bureau of Labor Statistics, *Wholesale Prices and Price Indexes*, (January 1971), pp. 33, 106-107.

[2] A relatively large set of monthly time series data may appear to contain much information because it contains so many data points. The amount of that information may be illusory, however. If the data are afflicted with monthly seasonality, then the observations may contain no more information than that contained in year-to-year averages distributed over the years from which the observations were drawn--in which case the investigator essentially has based his analysis on nothing more than repetitious seasonal patterns. The result may be statistically biased estimates. For further discussion, see Christopher A. Sims, "Seasonality in Regression," *Journal of the American Statistical Association*, Vol. 69 (September 1974), pp. 618-26.

[3] Albert Rees, "On Equilibrium in Labor Markets," *Journal of Political Economy*, Vol. 78 (March/April 1970), p. 309.

operations, and continued in monthly increments from the date of imposition.[1] Thus, in Models 1 and 3, REGDUM defines a separate linear time segment associated with EPA and OSHA regulations. If the coefficient of REGDUM were negative and statistically significant, one could infer that between the pre- and post-regulation periods something empirically significant occurred that had an unpropitious impact on the industry's output. However, it would be heroic to conclude from such evidence alone that the culprits were EPA and OSHA regulations.

Unfortunately, estimation of Models 2 and 4 had to be based upon even fewer monthly observations than those used in the estimation of Models 1 and 3. This latter data constraint was dictated by the method variable REG could be measured (see Appendix A). As a result of these data limitations, Models 2 and 4 had to be estimated on the basis of twenty-four monthly observations drawn from calendar years 1973 and 1974. Since this latter sample is relatively small, these estimates must be interpreted with caution.[2]

Another unfortunate aspect of our research is that in both the TSLS estimation of Model 2 and the ATSLS estimation of Model 4, the moment matrix of regressors would not invert because, undoubtedly, there exists considerable multicollinearity among these variables. Various procedures have been advanced for coping with this problem. These range from the Brundy and Jorgenson instrumental variables methods[3] to the mere deletion of one

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[1]This same technique has been employed in another context by Russell. See Louis B. Russell, *The Diffusion of Hospital Technologies: Some Econometric Evidence*, The Brookings Institution, Washington, D.C., n.d. (Mimeographed.)

[2]See n. 2, p. 12.

[3]James M. Brundy and Dale W. Jorgenson, "Efficient Estimation of Simultaneous Equations by Instrumental Variables," *Review of Economics and Statistics*, Vol. 53 (August 1971), pp. 207-224. Also see the following papers by Brundy and Jorgenson: "Consistent and Efficient Estimation of Systems of Simultaneous Equations," in *Frontiers in Econometrics*, ed. Paul Zarembka (New York: Academic Press, Inc., 1974), pp. 215-44; (continued on next page)

or more explanatory variables.[1] In our research, trial and error were used to identify which explanatory variables appeared to be linearly related--various explanatory variables were regressed on subsets of other explanatory variables--and on the basis of those results, WAGE1 was dropped from the supply equation and WAGE2 was dropped from the demand equation of each of these two models (i.e., Models 2 and 4).

The shortcomings of arbitrarily dropping explanatory variables from structural relations are well known and need not be repeated here.[2] Even though the estimates of the coefficients in Models 2 and 4 might not be precise, it was hoped that when considered along with the estimates of the coefficients of Models 1 and 3, they might shed some light on the question of whether or not EPA and OSHA regulations have had the impact on foundries that critics have alleged.

Three other matters require comment before we turn to this study's empirical results. First, it should be noted that Models 3 and 4 are based on the "quantitative method" of Fair and Jaffe,[3] which relies on the classical theory of price adjustment in a competitive market--price will rise when there is excess demand, and fall when there is excess supply. It has been argued, however, that there is no reason to believe that the optimizing behavior of economic agents will lead to a unique price level except in equilibrium; rather, a multiplicity of

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(contd) and "The Relative Efficiency of Instrumental Variables Estimation of Systems of Simultaneous Equations," *Annals of Economic and Social Measurement*, Vol. 3 (October 1974), pp. 679-700.

[1]Aigner recommends that such deletions be made according to a mean-square-error criterion. See Dennis J. Aigner, "Basic Econometrics" (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1971), p. 96.

[2]For example, see E. Malinvaud, "Statistical Methods of Econometrics," 2d ed. (Amsterdam: North-Holland Publishing Company, 1970), pp. 311-14; and Brundy and Jorgenson, *op. cit.*, "Efficient Estimation," pp. 216-18.

[3]Ray C. Fair and Dwight M. Jaffee, "Methods of Estimation for Markets in Disequilibrium," *Econometrica*, Vol. 40 (May 1972), pp. 497-514.

price levels will characterize a market that is in disequilibrium.[1] Thus in the price-adjustment equations of Models 3 and 4, Δy_{1t} is most appropriately viewed as the change in the *average* price level of castings between months t-1 and t.[2]

A second point which should be made explicit is the fact that the ATSLS estimates of the coefficients of Models 3 and 4 are not asymptotically efficient.[3] Any future research effort that investigates the issues examined herein might benefit from alternative (maximum likelihood) estimation techniques that have been devised by Amemiya.[4]

Finally, it should be noted that the four equations in Model 3 can be collapsed into two:[5]

$$QUAN_t = \alpha_{30} + \sum_{k=2}^{5} \alpha_{3k} x_{k,t-1} - (1/\gamma_3)G_t + \varepsilon_{3t}^{D} \tag{15}$$

$$QUAN_t = \beta_{30} + \sum_{j=2}^{5} \beta_{3j} z_{j,t-1} - (1/\gamma_3)H_t + \varepsilon_{3t}^{S}, \tag{16}$$

where

$$\begin{aligned} G_t &= \Delta(PRICE1_t/WPI_t) && \text{if } \Delta(PRICE1_t/WPI_t) > 0 \\ &= 0 && \text{otherwise} \\ H_t &= -\Delta(PRICE1_t/WPI_t) && \text{if } \Delta(PRICE1_t/WPI_t) < 0 \\ &= 0 && \text{otherwise.} \end{aligned}$$

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[1]Kenneth J. Arrow, "The Role of Price Adjustment Equations in Economic Theory," in Moses Abramovitz, *et al.*, *The Allocation of Economic Resources* (Stanford, Calif.: Stanford University Press, 1959), pp. 41-51.

[2]Cf. Kenneth J. Arrow and William M. Capron, "Dynamic Shortages and Price Rises: The Engineer-Scientist Case," *Quarterly Journal of Economics*, Vol. 73 (May 1959), pp. 292-308.

[3]Amemiya, *op. cit.*, "A Note," p. 760.

[4]*Idem.*, pp. 760-61.

[5]*Idem.*, p. 759.

Similarly, for Model 4 we have

$$QUAN_t = \alpha_{40} + \sum_{k=2}^{5} \alpha_{4k} x_{k,t-1} - (1/\gamma_4)G_t + \varepsilon_{4t}^{D} \qquad (17)$$

$$QUAN_t = \beta_{40} + \beta_{42} z_{2t}^{*} + \sum_{j=3}^{5} \beta_{4j} z_{j,t-1} - (1/\gamma_4)H_t + \varepsilon_{4t}^{S} , \qquad (18)$$

where G_t and H_t are as defined above.

C. EMPIRICAL FINDINGS AND CONCLUSIONS

Tables 1, 2, 3, and 4 present our estimates of the coefficients appearing in Models 1, 2, 3, and 4, respectively, as well as the estimated values of the coefficient of determination and Durbin-Watson statistic.

As regards *anticipated* results, if the allegations made by the critics of EPA and OSHA regulations are correct, then the coefficients of the regulation variables (REGDUM and REG) could be expected to be negative in sign, although such a sign would not necessarily provide an unambiguous indication that those regulations have had a deleterious effect on the industry. As mentioned above, a negative sign for REGDUM may actually be a surrogate for some phenomenon other than EPA and OSHA regulations that otherwise is not accounted for by Models 1 and 3, and that occurred for the first time in May 1971 (and continued to exist after May 1971). Furthermore, if EPA and OSHA regulations were so stringent that a mass exodus of firms occurred, aggregate industry expenditures on pollution abatement and safety equipment might have actually declined, even though such outlays made by each remaining firm rose. In that event, the parallel reduction in both the number of firms in the industry and the aggregate expenditures on pollution abatement and safety equipment would yield a coefficient of REG that was positive in sign. That event did not occur, however--at least the data employed in this study are inconsistent with that outcome.

Table 1. ESTIMATES FOR MODEL 1

Equation Number	Coefficients of Explanatory Variables[a]						Coefficient of Determination[b]	Durbin-Watson Statistic
(1)	PRICE1/WPI	WAGE2/WPI	COST/WPI	PRICE2/WPI	TREND	CONSTANT	R^2	d
	2922.847	90,874.216[c]	-675.812	-6744.104[c]	-12.532[c]	2539.305[c]	0.4984[c]	1.6646
	(1.7580)	(3.1872)	(-0.5524)	(-2.3254)	(-2.3254)	(3.0269)	(16.70)	
(2)	PRICE1/WPI	REGDUM	WAGE1/WPI	COST/WPI	TREND	CONSTANT	R^2	d
	-2340.537[c]	11.190[c]	21,147.584[c]	2232.535	-5.670	237.612	0.3555[c]	1.0737[c]
	(4.2040)	(2.2899)	(2.8825)	(1.8434)	(-1.4513)	(0.2877)	(9.71)	

[a] t values in parentheses.

[b] Adjusted for degrees of freedom; F value in parentheses.

[c] Significant at the .05 level.

Table 2. ESTIMATES FOR MODEL 2

Equation Number	Coefficients of Explanatory Variables[a]					Coefficient of Determination[b]	Durbin-Watson Statistic
(4)	PRICE1/WPI	COST/WPI	PRICE2/WPI	TREND	CONSTANT	R^2	d
	127.548	1949.455	-3538.638[c]	-18.657[c]	2362.836[c]	0.2715[c]	2.2351
	(0.0503)	(0.6114)	(-2.9122)	(-2.8117)	(3.3983)	(3.05)	
(5)	PRICE1/WPI	REG/WPI	COST/WPI	TREND	CONSTANT	R^2	d
	33,763.913	38.692	-35,783.962	40.956	-9079.618	0.0579	1.6289
	(0.5087)	(0.4868)	(-0.5169)	(0.4288)	(-0.4160)	(0.70)	

[a] t values in parentheses.

[b] Adjusted for degrees of freedom; F value in parentheses.

[c] Significant at the .05 level.

Table 3. ESTIMATES FOR MODEL 3

Equation Number	Coefficients of Explanatory Variables[a]						Coefficient of Determination[b]	Durbin-Watson Statistic
(15)	G	WAGE2/WPI	COST/WPI	PRICE2/WPI	TREND	CONSTANT	R^2	d
	-21,389.872[c]	12,022.068	1097.015	3738.402[c]	-4.322[c]	3270.501[c]	0.5570[c]	1.2805[c]
	(-2.0805)	(0.6681)	(1.3667)	(5.2022)	(-5.2286)	(3.2140)	(21.12)	
(16)	H	REGDUM	WAGE1/WPI	COST/WPI	TREND	CONSTANT	R^2	d
	14,848.292	-3.108	-15,192.673	-1372.084	0.112	2723.001[c]	0.4253[c]	1.0853[c]
	(1.7030)	(-0.6531)	(-0.8837)	(-1.5885)	(0.0292)	(5.5678)	(12.84)	

[a] t values in parentheses.

[b] Adjusted for degrees of freedom; F value in parentheses.

[c] Significant at the .05 level.

Table 4. ESTIMATES FOR MODEL 4

Equation Number	Coefficients of Explanatory Variables[a]					Coefficient of Determination[b]	Durbin-Watson Statistic
(17)	G	COST/WPI	PRICE2/WPI	TREND	CONSTANT	R^2	d
	-1878.066	2057.005	3591.699[c]	-17.645[c]	2438.938[c]	0.2724[c]	2.1801
	(-0.1625)	(1.8590)	(2.9417)	(-2.2664)	(2.8639)	(3.06)	
(18)	H	REG/WPI	COST/WPI	TREND	CONSTANT	R^2	d
	88,104.304	-20.271	-9218.622	27.850	13,938.305	0.0159	1.7166
	(1.2759)	(-1.3548)	(-1.3509)	(0.9546)	(1.4732)	(1.09)	

[a] t values in parentheses.

[b] Adjusted for degrees of freedom; F value in parentheses.

[c] Significant at the .05 level.

Regarding the other variables, on theoretical grounds it was expected that the coefficient of PRICE1/WPI in equations (1) and (3) would be negative, while in equations (2) and (4) it would be positive. In equations (1), (3), (15) and (17), no *a priori* sign expectation could be attached to the coefficients of WAGE2/WPI and COST/WPI--if the labor (non-labor) input and costings were substitutes in production, the coefficient of WAGE2/WPI (COST/WPI) would be positive; if they were complements, it would be negative. On *a priori* grounds alone there was no way to anticipate the sign of TREND's coefficient in any of the equations. However, theoretical considerations suggest that the coefficient of PRICE2/WPI should be positive in sign. Finally, given the price adjustment hypothesis, $\gamma > 0$--i.e., the greater excess demand (supply) is, the greater the increase (decrease) in price will be--and given the definitions of the variables G and H, we would anticipate that the coefficient of G in equations (15) and (17) would be negative, while the coefficient of H in equations (16) and (18) would be positive.[1]

As compared to the anticipated results, those presented in Tables 1, 2, 3, and 4 are disappointing. The signs of several of the equilibrium models' coefficients (Tables 1 and 2) are "wrong." That result, coupled with the knowledge that considerable economic turbulence existed in the early 1970s, casts doubt on the assumption of equilibrium in the market for castings during that period. Inspection of the estimates obtained for the disequilibrium models' coefficients (Tables 3 and 4) reveals more tenable results. Although in some cases the coefficients are not significantly different from zero at the .05

[1]In fact, the true values of those two coefficients should be numerically equal, although opposite in sign. Preliminary results not reported here indicate that, unlike the ATSLS estimation scheme, Amemiya's maximum likelihood technique (which is an iterative procedure) will converge to a solution value for γ which is unique. The latter procedure is described in Amemiya, *op. cit.*, "A Note," pp. 760-61.

level, at least their signs are consistent with our theoretical expectations. In particular, it should be noted that the coefficients of REGDUM and REG in equations (16) and (18) respectively, are negative, although they are not statistically significant.

In conclusion, this study has undertaken an empirical examination of the economic impact of EPA and OSHA regulations on the ferrous foundry industry. The value of any further investigation of this issue would be enhanced considerably by access to a greater quantity and quality of data than were available for our research. In addition to these data requirements, any future research on the economic effects of EPA and OSHA regulations on the foundries should endeavor to implement the various econometric suggestions that have been put forth herein.

APPENDIX A

VARIABLES AND DATA SOURCES

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Current Endogenous Variables

QUAN = Thousands of (short) tons of gray iron, malleable iron, and steel castings shipments for sale. (Source: *Survey of Current Business*, various issues.)

PRICE1 = Wholesale price index for foundry and forge shop products. (Source: *Wholesale Prices and Price Indexes*, various issues.)

ΔPRICE1 = PRICE1 - PRICE1L (PRICE1L defined below.)

Predetermined Variables (Exogenous or Lagged Endogenous)

REG = Expenditures by iron and steel foundries on pollution abatement and safety equipment. (Source: See below.)

REGDUM = 0 for each month beginning with January 1970 and ending with April 1974, 1 for May 1974, 2 for June 1974, and so forth.

COST = Wholesale price index for industrial commodities. (Source: *Survey of Current Business*, various issues.)

WAGE1 = Average hourly earnings of production or non-supervisory workers in iron or steel foundries. (Source: *Employment and Earnings*, various issues.)

WAGE2 = Average hourly earnings of production or non-supervisory workers in manufacturing industries. (Source: *Survey of Current Business*, various issues.)

PRICE2 = Wholesale price index for producer-finished goods. (Source: *Survey of Current Business*, various issues.)

TREND = Index for month in which observations were made. (TREND equals 1,2,...,82 for Models 1 and 3, and 1,...,24 for Models 2 and 4).

PRICE1L = PRICE1 lagged one month.

Deflator

WPI = Wholesale price index. (Source: *Survey of Current Business*, various issues.)

Definition of REG

To derive REG, we make use of the symbols listed below.

t: Month index: t = 1 for Jan., t = 2 for Feb., etc.

j: Year index: j = 1 for 1973, j = 2 for 1974.

EM_{jt}: Number of production workers employed by iron and steel foundries during month t in year j. (Source: *Employment and Earnings*, various issues.)

EM_j: Average number of production workers employed by iron and steel foundries for year j. (Source: *Employment and Earnings*, various issues.)

TCE_j: New capital expenditures made by iron and steel foundries during year j. (Source: *Current Industrial Reports*, 1973 and 1974.)

$PACE_j$: Expenditures made by iron and steel foundries on pollution abatement equipment during year j. (Source: *Current Industrial Reports*, 1973 and 1974.)

$PAOME_j$: Pollution abatement operating and maintenance expenditures made by iron and steel foundries during year j. (Source: *Current Industrial Reports*, 1973 and 1974.)

SEE_j: Safety equipment expenditures per production worker by iron and steel foundries during year j. (Source: Bolt, Beranek and Newman report,[1] p. F-29.)

No data exist on the next variables--but they will be eliminated during the course of the derivation of REG.

TCE_{jt}: New capital expenditures made by iron and steel foundries during month t in year j.

$EQUIP_{jt}$: Safety equipment expenditures made by iron and steel foundries during month t in year j.

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[1]Bolt, Beranek and Newman, Inc., *Impact of Noise Control at the Workplace*, Report No. 2671, submitted to the U.S. Department of Labor, Office of Standards, January 1, 1974.

$EQUIP_j$: Safety equipment expenditures made by iron and steel foundries during year j.

REG_{jt}: Expenditures made for pollution abatement efforts and safety equipment by iron and steel foundries during month t in year j.

Define

$$PAE_{jt} = PACE_{jt} + PAOME_{jt},\ PAE_j = PACE_j + PAOME_j. \tag{A.1}$$

Assume that the ratio of pollution abatement and safety equipment expenditures to total new capital expenditures during month t of year j equals the ratio of pollution abatement and safety equipment expenditures to total new capital expenditures for year j:

$$(PAE_{jt} + EQUIP_{jt})/TCE_{jt} = (PAE_j + EQUIP_j)/TCE_j, \tag{A.2}$$

or

$$PAE_{jt} + EQUIP_{jt} = TCE_{jt}(PAE_j + EQUIP_j)/TCE_j. \tag{A.3}$$

Also, assume that the ratio of total new capital expenditures per production worker during month t of year j equals the ratio of total new capital expenditures per production worker during year j:

$$TCE_{jt}/EM_{jt} = TCE_j/EM_j, \tag{A.4}$$

or

$$TCE_{jt} = EM_{jt}(TCE_j/EM_j). \tag{A.5}$$

Substitute (A.5) into (A.3):

$$PAE_{jt} + EQUIP_{jt} = (PAE_j + EQUIP_j)(EM_{jt}/EM_j). \tag{A.6}$$

By definition,

$$SEE_j = EQUIP_j/EM_j, \tag{A.7}$$

which implies

$$EQUIP_j = (EM_j)(SEE_j). \tag{A.8}$$

Substitute (A.8) into (A.6):

$$PAE_{jt} + EQUIP_{jt} = [PAE_j + (EM_j)(SEE_j)](EM_{jt}/EM_j). \qquad (A.9)$$

Define

$$REG_{jt} = PAE_{jt} + EQUIP_{jt}. \qquad (A.10)$$

Substitute (A.10) and the second equation of (A.1) into (A.9):

$$REG_{jt} = \left(PACE_j + PAOME_j + [(EM_j)(SEE_j)]\right)\left(EM_{jt}/EM_j\right). \qquad (A.11)$$

Data on REG were generated via the latter equation.

APPENDIX B

MATHEMATICAL APPENDIX

MATHEMATICAL APPENDIX

In effect, EPA and OSHA regulations require firms in the ferrous foundry industry to purchase pollution abatement and safety equipment. An inevitable consequence of the typical foundry's best-practice technology is the joint production of saleable output, air pollutants, and worker injuries. For simplicity, however, we shall ignore the joint production that characterizes the foundry's operations. While a model of production that formally accounts for joint outputs would be more descriptively realistic of a foundry's operations, it would make the analysis that follows unduly complicated.[1]

This appendix makes use of theorems advanced in an important article by Ferguson and Saving,[2] and corollaries to those propositions established by Maurice.[3] We shall assume that the ferrous foundry industry is a perfectly competitive one, and that the typical foundry purchases its inputs in perfectly competitive resource markets. Thus, the firm takes the market prices of resources and its saleable output as given.

It now is convenient to introduce three elasticities: the *own-price elasticity of demand* for the industry's product

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[1]A model of joint production is presented in David K. Whitcomb, "Externalities and Welfare" (New York: Columbia University Press, 1972), pp. 22-46; and in Daniel C. Vandermeulen, "Linear Economic Theory" (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1971), pp. 225-47.

[2]C. E. Ferguson and Thomas R. Saving, "Long-Run Scale Adjustments of a Perfectly Competitive Firm and Industry," *American Economic Review*, LIX (December 1969), pp. 774-83.

[3]Charles Maurice, "Factor-Price Changes, Profit, and Long-Run Equilibrium," *Western Economic Journal*, IX (March 1971), pp. 64-77.

($\eta_D > 0$); the firm's *elasticity of marginal cost* ($\eta_{MC} > 0$); and the i^{th} factor's *expenditure elasticity* (η_i, undetermined in sign).[1] Let the typical firm's production function be defined as

$$q = f(x_1,x_2), \tag{1}$$

where q is the maximum quantity of output that the firm can produce if it employs x_1 units of the first resource, (say) capital, and x_2 units of the second resource, (say) labor. The firm's total cost of production is defined as

$$c = \sum_{i=1}^{2} r_i x_i, \tag{2}$$

where r_i is the i^{th} input's market price.

Consider now the problem of minimizing total cost, subject to a specified level of output (q_0). The Lagrangian for this problem is

$$L = \sum_{i=1}^{n} r_i x_i + \mu\left[q_0 - f(x_1,x_2)\right] . \tag{3}$$

The Lagrange multiplier, μ, is the firm's marginal cost:[2]

$$\mu = \partial c/\partial q. \tag{4}$$

In order for the profit-maximizing, perfectly competitive firm to be in long-run equilibrium, the following condition must hold:

$$p = \mu = c/q, \tag{5}$$

where p denotes the market-determined price of the industry's product. The aggregate quantity supplied of this product is

[1]For a precise definition of these elasticities, see Maurice, *op. cit.*, "Factor-Price Changes," pp. 66-67.

[2]For proof, see Paul A. Samuelson, "Foundations of Economic Analysis" (Cambridge: Harvard University Press, 1947), pp. 65-66.

$$Q_S = Nq, \qquad (6)$$

where q is the typical firm's output and N denotes the number of firms belonging to the industry. (For simplicity, we assume that N is continuously differentiable.)

Consider now a parametric variation in the market price of the i^{th} resource. Maurice has shown that

$$\partial q/\partial r_i = (1/\mu)x_i(1 - \eta_i)/\eta_{MC},^1 \qquad (7)$$

and

$$\partial N/\partial r_i = (Nx_i/c)\left[\eta_D\eta_{MC} - (1 - \eta_i)\right]/\eta_{MC}.^2 \qquad (8)$$

Maurice has also shown that when the firm has maximized profit, but the industry has not attained long-run equilibrium, then instead of (7) we have

$$\partial q/\partial r_i = q\eta_i\eta_D x_i/c(1 - \eta_D\eta_{MC}).^3 \qquad (9)$$

In other words, (7) holds only when both firm and industry are in long-run equilibrium.

If we differentiate both sides of (6) with respect to r_i, we obtain

$$\partial Q_S/\partial r_i = N(\partial q/\partial r_i) + q(\partial N/\partial r_i). \qquad (10)$$

Substitution of (7) and (8) into (10) yields

$$\partial Q_S/\partial r_i = Nx_i\eta_D/\mu c. \qquad (11)$$

Finally, if we substitute the second equation in (5) into (11), we obtain the desired expression for the shift in the industry's long-run supply curve occasioned by a parametric variation in r_1:

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[1]Cf. Maurice, *op. cit.*, "Factor-Price Changes," equation (12), p. 67.
[2]This equation follows immediately from *idem.*, equation (22), p. 73.
[3]*Idem.*, equation (13), p. 68.

$$\partial Q_S/\partial r_i = qNx_i\eta_D/c < 0. \tag{12}$$

Since q, N, x_i, c > 0 and $\eta_D < 0$, it follows that $\partial Q_S/\partial r_i < 0$.

The foregoing theoretical results have empirical implications. We may infer from (12) that, other things being equal, an increase in the market price of the i^{th} resource will cause a reduction in the output of the ferrous foundry industry, *irrespective* of whether that resource is an inferior ($\eta_i < 0$), normal ($0 < \eta_i < 1$), or superior ($\eta_i > 1$) input. Further, since (by assumption) η_{MC}, x_i, $\mu > 0$, it follows from (7) that the long-run firm supply response to a resource price change depends on that resource's classification. If it is an inferior or normal input, then $\partial q/\partial r_i > 0$, i.e., an increase in a resource price will actually cause the competitive firm to increase its long-run optimal scale of operations. On the other hand, if the resource whose price has risen is a superior input, then $\partial q/\partial r_i < 0$. More generally, we have the following result:

$$\partial q/\partial r_i < 0 \ (> 0) \text{ only if } \eta_i > 1 \ (\leqq 1). \tag{13}$$

Thus, in the case of a *non-superior* input (i.e., $\eta_i \leqq 1$), (10) and (12) imply that $\partial N/\partial r_i < 0$. That is, if the resource whose price has increased is a non-superior input, then the long-run reduction in industry supply of saleable output is attributable solely to the exodus of firms from the industry. In general, however, the impact of a resource price change on the number of firms comprising the industry cannot be determined on *a priori* grounds alone. Maurice has shown that

$$\partial N/\partial r_i = (Nx_i/c)\left[\left\{\eta_D\eta_{MC} - (1 - \eta_i)\right\}\eta_{MC}\right],^1 \tag{14}$$

where N is the optimal number of firms (i.e., the number of firms necessary for long-run equilibrium to exist in the market

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[1]This equation follows immediately from *idem.*, equation (22), p. 73.

for the industry's product when the typical firm is producing at minimum long-run average cost[1]). Since N, x_i, c, $\eta_{MC} > 0$ and $\eta_D < 0$, (14) implies that

$$\partial N/\partial r_i \gtrless 0 \quad \text{according as} \quad (1 - \eta_i) \lessgtr \eta_D \eta_{MC}.^2 \qquad (15)$$

The long-run impact of a resource price change of the typical firm's profit (π) can be shown to be

$$\partial \pi/\partial r_i = x_i \left[\left\{ \eta_i/(1 - \eta_D \eta_{MC}) \right\} - 1 \right].^3 \qquad (16)$$

Since x_i, $\eta_{MC} > 0$ and $\eta_D < 0$, (16) implies that

$$\partial \pi/\partial r_i \gtrless 0 \quad \text{according as} \quad (1 - \eta_i) \lessgtr \eta_D \eta_{MC}.^4 \qquad (17)$$

Comparison of (15) and (17) reveals that conditions causing profit (loss) after a resource price change also cause the optimal number of firms comprising the industry to increase (decrease).[5] Note that if we divide (14) by (16), we obtain

$$(\partial N/\partial r_i)/(\partial \pi/\partial r_i) = N(1 - \eta_D \eta_{MC})/c\eta_{MC}.^6 \qquad (18)$$

Since N, c, $\eta_{MC} > 0$ and $\eta_D < 0$, (18) implies that the number of firms in the industry varies directly with profit:

$$(\partial N/\partial r_i)/(\partial \pi/\partial r_i) > 0. \qquad (19)$$

To recapitulate, we have established that on *a priori* grounds alone the effect of a resource price change on the number of firms in the industry cannot be determined; that question is strictly an empirical one. However, on theoretical

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[1]*Idem.*, p. 72.

[2]Cf. *idem.*, statement (23), p. 73.

[3]*Idem.*, equation (16), p. 69.

[4]*Idem.*, statement (17), p. 69.

[5]*Idem.*, p. 73.

[6]*Idem.*, equation (24), p. 73.

grounds it can be predicted that the number of firms will vary directly with profits in the industry. Let us now turn to an examination of the impact of EPA and OSHA regulations on the supply behavior of the ferrous foundry industry as a whole, as well as the typical firm belonging to that industry.

Suppose that the effect of initial EPA and OSHA regulations is to require the typical foundry to purchase t dollars worth of fixed quantities of pollution abatement and safety equipment. This initial expenditure requirement is tantamount to the imposition of a lump-sum tax.

Prior to the imposition of the initial expenditure requirement, the firm's profit can be expressed as

$$\pi = pf(x_1,x_2) - \sum_{i=1}^{2} r_i x_i . \tag{20}$$

The first-order conditions for profit maximization are

$$\partial\pi/\partial x_i = pf_i - r_i = 0 \qquad (i = 1,2), \tag{21}$$

where $f_i \equiv \partial q/\partial x_i$, or

$$pf_i = r_i, \qquad (i = 1,2), \tag{22}$$

i.e., the marginal revenue product of each input must equal the market price of that input. The corresponding second-order conditions are

$$\partial^2\pi/\partial x_i^{\,2} = pf_{ii} < 0 \qquad (i = 1,2),$$

$$\begin{vmatrix} \partial^2\pi/\partial x_i^{\,2} & \partial^2\pi/\partial x_1\partial x_2 \\ \partial^2\pi/\partial x_2\partial x_1 & \partial^2\pi/\partial x_2^{\,2} \end{vmatrix} = p^2F^* > 0, \tag{23}$$

where $f_{ij} \equiv \partial^2 f/\partial x_i \partial x_j$ and

$$F^* = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} = f_{11}f_{22} - f_{12}f_{21} = f_{11}f_{22} - f_{12}^2.[1] \qquad (24a)$$

Since $p^2 > 0$, (23) implies that

$$F^* > 0. \qquad (24b)$$

It is easy to prove that

$$c = \kappa(q, r_1, r_2),[2] \qquad (25)$$

where κ is the firm's total cost function. Thus, an alternative formulation of the firm's profit function is

$$\pi = pq - \kappa(q, r_1, r_2). \qquad (26)$$

If we hold input prices fixed, the first-order condition for profit maximization is

$$\partial\pi/\partial q = p - \partial c/\partial q = 0, \qquad (27)$$

or

$$p = \partial c/\partial q, \qquad (28)$$

i.e., the firm's marginal revenue must equal its marginal cost. The corresponding second-order condition is

$$\partial^2\pi/\partial q^2 = -\partial^2 c/\partial q^2 < 0. \qquad (29)$$

After the imposition of the initial expenditure requirement, the typical foundry's profit function may be expressed as

$$\begin{aligned} \pi &= pf(x_1, x_2) - \sum_{i=1}^{2} r_i x_i - t \\ &= pq - \kappa(q, r_1, r_2) - t. \end{aligned} \qquad (30)$$

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[1]For a proof of the theorem that $f_{12} = f_{21}$, see Angus E. Taylor, "Advanced Calculus" (Waltham, Mass.: Blaisdell Publishing Company, 1955), pp. 220-21.

[2]For a derivation, see Samuelson, *op. cit.*, "Foundations," pp. 58-59.

Since $\underline{t}$ is--from the firm's perspective--a constant, it is clear that the first- and second-order conditions for the maximization of (30) are identical to those for the maximization of (20) and (26). Thus, the effect of the initial requirement to purchase fixed amounts of pollution abatement and safety equipment is to leave unaltered the foundry's level of production, provided that $\pi \geqq 0$. The initial expenditure requirement affects only the firm's decision whether to continue or close down operations. If the initial required expenditure exceeds maximum net revenue (i.e., if $t > pq - c$), then the firm will be forced to shut down. In the latter event the number of firms in the industry will decline, and the market supply curve for the industry's saleable output will shift to the left.

The foregoing results regarding the impact of EPA and OSHA regulations can be established rigorously by considering an infinitesimal increase in $\underline{t}$. Consider the problem of maximizing output, subject to a fixed level of total cost. The Lagrangian for this problem is

$$\Lambda = f(x_1, x_2) + \lambda\left(c - \sum_{i=1}^{2} r_i x_i - t\right). \qquad (31)$$

The first-order conditions necessary for the existence of a relative maximum of output are

$$\partial\Lambda/\partial x_i = f_i - \lambda r_i = 0 \qquad (i = 1,2)$$

$$\partial\Lambda/\partial\lambda = c - \sum_{i=1}^{2} r_i x_i - t = 0, \qquad (32)$$

and the corresponding second-order condition is

$$H_q = \begin{vmatrix} -f_{11} & -f_{12} & -r_2 \\ -f_{21} & -f_{22} & -r_2 \\ -r_1 & -r_2 & 0 \end{vmatrix} > 0. \qquad (33)$$

Let us rewrite system (32) as the following set of implicit functions:[1]

$$\psi^k(x_1, x_2, \lambda, r_1, r_2, c, t) = 0 \qquad (k = 1,2,3). \tag{34}$$

The Jacobian of this system is

$$J_q = \begin{vmatrix} \partial\psi^1/\partial x_1 & \partial\psi^1/\partial x_2 & \partial\psi^1/\partial\lambda \\ \partial\psi^2/\partial x_1 & \partial\psi^2/\partial x_2 & \partial\psi^2/\partial\lambda \\ \partial\psi^3/\partial x_1 & \partial\psi^3/\partial x_2 & \partial\psi^3/\partial\lambda \end{vmatrix}. \tag{35}$$

Comparison of (33) and (35) reveals that $J_q = H_q > 0$. Since this Jacobian is nonvanishing, we may appeal to the implicit function theorem and rewrite the implicit functions defined by (34) as the following set of explicit functions:

$$\begin{aligned} x_i &= \psi^i(r_1, r_2, c, t) \qquad (i - 1,2) \\ \lambda &= \psi^3(\bullet). \end{aligned} \tag{36}$$

Finally, consider the problem of minimizing total cost, subject to a specified level of output. This problem's Lagrangian is

$$L = \sum_{i=1}^{2} r_i x_i + t + \mu\left[q - f(x_1,x_2)\right]. \tag{37}$$

It can be shown that the Lagrange multiplier, μ, which is undetermined in sign, is the firm's marginal cost, assumed to be positive:

$$\kappa'(q) = \mu = \partial c/\partial q > 0. \tag{38}$$

The first-order conditions that are necessary for the existence of a relative minimum of total cost are

$$\begin{aligned} \partial L/\partial x_i &= r_i - \mu f_i = 0 \qquad (i = 1,2) \\ \partial L/\partial \mu &= q - f(x_1,x_2) = 0. \end{aligned} \tag{39}$$

[1]The mathematical results of this paragraph are discussed in Taylor, *op. cit.*, "Advanced Calculus," pp. 238-51.

The corresponding second-order condition is

$$H_c = \begin{vmatrix} -\mu f_{11} & -\mu f_{12} & -f_1 \\ -\mu f_{21} & -\mu f_{22} & -f_2 \\ -f_1 & -f_2 & 0 \end{vmatrix} < 0. \qquad (40)$$

It can be shown that

$$H_c = (-1/\mu)H_q. \qquad (41)$$

Let us rewrite system (39) as a set of implicit functions:

$$\Xi^k(x_1, x_2, \mu, r_1, r_2, q, t) = 0 \qquad (k = 1,2,3). \qquad (42)$$

The Jacobian of this system is

$$J_c = \begin{vmatrix} \partial\Xi^1/\partial x_1 & \partial\Xi^1/\partial x_2 & \partial\Xi^1/\partial\mu \\ \partial\Xi^2/\partial x_1 & \partial\Xi^2/\partial x_2 & \partial\Xi^2/\partial\mu \\ \partial\Xi^3/\partial x_1 & \partial\Xi^3/\partial x_2 & \partial\Xi^3/\partial\mu \end{vmatrix}. \qquad (43)$$

Comparison of (40) and (43) shows that $J_c = H_c < 0$. Hence, we may invoke the implicit function theorem and rewrite system (40) as the following set of explicit functions:

$$x_i = \xi^i(r_1, r_2, q, t) \qquad (i = 1,2)$$
$$\mu = \xi^3(\bullet). \qquad (44)$$

If we solve the first two equations of (32) for λ, we obtain

$$\lambda = f_i/r_i \qquad (i = 1,2). \qquad (45)$$

Similarly, we may solve the first two equations of (39) for μ:

$$\mu = r_i/f_i \qquad (i = 1,2). \qquad (46)$$

In view of (38), (45), and (46), we have

$$1/\lambda = \mu = \partial c/\partial q. \qquad (47)$$

Substitution of (47) into (28) provides an alternative statement of the condition for long-run equilibrium for the competitive firm:

$$p = 1/\lambda. \tag{48}$$

In order for long-run industry equilibrium to exist, one condition that must be satisfied is that the typical firm in the industry must have zero profit. This latter condition implies that the firm's average revenue equals its average cost:

$$p = c/q = \left(t + \sum_{i=1}^{2} r_i x_i\right) \Big/ f(x_1, x_2). \tag{49}$$

Thus, the existence of long-run competitive equilibrium is implied by the equality of average cost and marginal cost:

$$\left(t + \sum_{i=1}^{2} r_i x_i\right) \Big/ f(x_1, x_2) = 1/\lambda, \tag{50}$$

or, equivalently,

$$f(x_1, x_2) - \lambda\left(\sum_{i=1}^{2} r_i x_i + t\right) = 0. \tag{51}$$

Consider now a parametric variation in <u>t</u>. Specifically, let us substitute (36) into the first two equations of (32), and then differentiate the resultant expressions with respect to <u>t</u>:[1]

$$f_{i1}(\partial x_1/\partial t) + f_{i2}(\partial x_2/\partial t) = 0 \qquad (i = 1,2). \tag{52}$$

Next, substitute (36) into (51), and differentiate with respect to <u>t</u>:

$$\sum_{i=1}^{2} f_i\left(\frac{\partial x_i}{\partial t}\right) - \sum_{i=1}^{2} \lambda r_i\left(\frac{\partial x_i}{\partial t}\right) - \left(\sum_{i=1}^{2} r_i x_i\right)\frac{\partial \lambda}{\partial t} - \lambda - t\frac{\partial \lambda}{\partial t} = 0. \tag{53}$$

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[1]The discussion that follows parallels Ferguson and Saving, *op. cit.*, "Long-Run Scale Adjustments," pp. 775-76.

Upon collecting like terms, we obtain:

$$\sum_{i=1}^{2}(f_i - \lambda r_i)(\partial x_i/\partial t) - \left(\sum_{i=1}^{2} r_i x_i + t\right)(\partial\lambda/\partial t) = \lambda. \tag{54}$$

Substitution of (32) into (54) yields

$$-c(\partial\lambda/\partial t) = \lambda. \tag{55}$$

Evaluated at the optimum, the system comprised by (52) and (55) consists of three linear equations in three unknowns. Let us apply Cramer's rule[1] to this system and solve for $\partial x_1/\partial t$:

$$\partial x_1/\partial t = D_1/\tilde{F} = 0, \tag{56}$$

because

$$\tilde{F} = \begin{vmatrix} f_{11} & f_{12} & 0 \\ f_{21} & f_{22} & 0 \\ 0 & 0 & -c \end{vmatrix} = -cF^* < 0, \tag{57}$$

where F^* is defined by (24), and

$$D_1 = \begin{vmatrix} 0 & f_{12} & 0 \\ 0 & f_{22} & 0 \\ \lambda & 0 & -c \end{vmatrix} = 0. \tag{58}$$

Similarly, if we solve for $\partial x_2/\partial t$, we obtain

$$\partial x_2/\partial t = D_2/\tilde{F} = 0, \tag{59}$$

since

$$D_2 = \begin{vmatrix} f_{11} & 0 & 0 \\ f_{21} & 0 & 0 \\ 0 & \lambda & -c \end{vmatrix} = 0. \tag{60}$$

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[1]This rule is discussed in G. Hadley, "Linear Algebra" (Reading, Mass.: Addison-Wesley Publishing Company, Inc., 1961), pp. 166-67.

Finally, we may solve for $\partial\lambda/\partial t$:

$$\partial\lambda/\partial t = D_3/\tilde{F}, \tag{61}$$

where

$$D_3 = \begin{vmatrix} f_{11} & f_{12} & 0 \\ f_{21} & f_{22} & 0 \\ 0 & 0 & \lambda \end{vmatrix} = \lambda F^*. \tag{62}$$

If we substitute (57) and (62) into (61), we obtain

$$\partial\lambda/\partial t = -\lambda/c < 0. \tag{63}$$

At last we are in a position to show that the typical firm's output will remain invariant with respect to the required expenditure, provided that long-run profit does not become negative.[1] Substitute the first two equations in (36) into (1) and then differentiate with respect to $\underline{t}$:

$$\partial q/\partial t = f_1(\partial x_1/\partial t) + f_2(\partial x_2/\partial t). \tag{64}$$

Substitute (56) and (59) into (64):

$$\partial q/\partial t = 0. \tag{65}$$

Having proved that the typical firm's long-run scale of operations will remain unchanged by the imposition of a required expenditure of $\underline{t}$ dollars (provided that profit remains non-negative), let us now consider the impact of this requirement on the optimal number ($\underline{N}$) of firms in the industry. A second condition that must be satisfied if long-run industry equilibrium is to exist is, of course, that aggregate quantity demanded (Q_D) must be equal to aggregate quantity supplied (Q_S). That is, in view of equation (6), this requirement is

$$Q_D = Nq. \tag{66}$$

[1]Recall that the derivations of (56) and (59) were based, in part, on the requirement that (49) be satisfied, i.e., that long-run profit equal zero. If that profit is negative, then the firm will shut down, i.e., q = 0.

If we solve (66) for N, and then differentiate with respect to t, we obtain

$$\partial N/\partial t = (\partial Q_D/\partial t)/q - Q_D(\partial q/\partial t)/q^2. \tag{67}$$

Substitute (65) into (67):

$$\partial N/\partial t = (\partial Q_D/\partial t)/q. \tag{68}$$

Before we can determine the sign of expression (68), we must define the own-price elasticity of demand for the industry's product, η_D. Let h be the inverse market demand function for the industry's product:

$$p = h(Q_D), \qquad h'(Q_D) < 0.^1 \tag{69}$$

Then

$$\eta_D = p/Q_D h'(Q_D).^2 \tag{70}$$

Substitute (66) into (70):

$$\eta_D = p/Nqh'(Q_D). \tag{71}$$

Next, differentiate (69) with respect to t:

$$\partial p/\partial t = (\partial p/\partial Q_D)(\partial Q_D/\partial t) = h'(Q_D)(\partial Q_D/\partial t). \tag{72}$$

If we solve this latter equation for $\partial Q_D/\partial t$, we obtain

$$\partial Q_D/\partial t = (\partial p/\partial t)/h'(Q_D). \tag{73}$$

To obtain an expression for the numerator of the ratio appearing on the right-hand side of (73), we differentiate (48) with respect to t:

$$\partial p/\partial t = (-1/\lambda^2)(\partial \lambda/\partial t). \tag{74}$$

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[1]Maurice, *op. cit.*, "Factor-Price Changes," equation (7), p. 66.
[2]*Idem.*, equation (9), p. 67.

Substitute (73) into (72):

$$\partial Q_D/\partial t = -(\partial\lambda/\partial t)/\lambda^2 h'(Q_D). \tag{75}$$

Then substitute this latter result into (68):

$$\partial N/\partial t = -(\partial\lambda/\partial t)/q\lambda^2 h'(Q_D). \tag{76}$$

If we multiply both numerator and denominator of the ratio on the right-hand side of (76), by N, and then substitute (63) into the resultant equation, we obtain

$$\partial N/\partial t = (Nx_1/c)/Nq\lambda h'(Q_D). \tag{77}$$

Substitute (48) into (77):

$$\partial N/\partial t = (Nx_1/c)\left[p/Nqh'(Q_D)\right]. \tag{78}$$

Finally, substitute (71) into (78):

$$\partial N/\partial t = \eta_D(Nx_1/c) < 0. \tag{79}$$

Since N, x_1, c > 0 and $\eta_D < 0$, it is clear that an infinitesimal increase in t will reduce the optimal number of firms in the industry.

We may use the foregoing results to theoretically examine the industry supply response to the required expenditure on pollution abatement and safety equipment. If we differentiate (6) with respect to t, we obtain

$$\partial Q_S/\partial t = N(\partial q/\partial t) + q(\partial N/\partial t). \tag{80}$$

Substitute (65) and (79) into (80):

$$\partial Q_S/\partial t = \eta_D(qNx_1/c) < 0. \tag{81}$$

Since q, N, x_1, c > 0 and $\eta_D < 0$, (81) reveals that the required expenditure will reduce aggregate quantity supplied by the industry, i.e., the market supply curve for the industry's product will shift to left.

The market supply of castings can be obtained by aggregating the supply functions of the individual foundries. To obtain the typical foundry's product supply curve, we assume that the firm's objective is to maximize its profit function, which is defined by (20). The first-order and second-order conditions for this problem are expressed by (22) and (23), respectively. Let us rewrite (22) as the following set of implicit functions:

$$\Omega_1(x_1,x_2,p,r_1,r_2) = 0$$

$$\Omega_2(\bullet) = 0. \tag{82}$$

The Jacobian of this system is

$$J_\pi = \begin{vmatrix} \partial\Omega_1/\partial x_1 & \partial\Omega_1/\partial x_2 \\ \partial\Omega_2/\partial x_1 & \partial\Omega_2/\partial x_2 \end{vmatrix}. \tag{83}$$

Comparison of (23), (24), and (83) reveals that $J_\pi = p_2F^* > 0$. Since $J_\pi \neq 0$, we may appeal to the implicit function theorem and rewrite equations (82) as the following set of explicit *input demand functions:*

$$x_1 = \omega_1(r_1,r_2,p)$$

$$x_2 = \omega_2(\bullet). \tag{84}$$

Substitution of (84) into (1) yields an equation that defines a composite function, γ:

$$q = f\left[\omega_1(r_1,r_2,p),\ \ \omega_2(r_1,r_2,p)\right] = \gamma(r_1,r_2,p). \tag{85}$$

The graph of γ is a hyperplane in input price, product price, product quantity space. The traditional output supply curve, however, is plotted in product quantity, product price space. Therefore, let us hold input prices fixed and define the product supply function <u>s</u> in the following way.

$$q = \gamma(p|r_1,r_2 = \text{constant}) = s(p). \tag{86}$$

The graph of $\underline{s}$ is the perfectly competitive, profit maximizing firm's product supply curve. An important relationship exists between this output supply curve and the firm's long-run marginal cost curve. The latter is the graph of the function $\kappa'(q)$, which is defined by equation (38). Let us rewrite the profit maximization first-order condition (28) as

$$p = \kappa'(q). \tag{87}$$

The firm's output supply function, $\underline{s}$, and its marginal cost function, κ', are *inverse* to each other.[1] That is

$$q = \kappa'^{-1}(p) = s(p). \tag{88}$$

Thus, for values of q for which the product's market price is at least as great as the firm's long-run average cost of production,[2] the firm's long-run marginal cost curve and its product supply curve are the "inverse" of one another.[3] This relationship is important for two reasons. First, the second-order condition for profit maximization requires the firm's long-run marginal cost curve to be rising.[4] Thus, assuming that there exist

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[1]The requirements for the existence of an inverse function are discussed in Tom M. Apostol, "Mathematical Analysis" (Reading, Mass.: Addison-Wesley Publishing Company, 1957), p. 29.

[2]Of course, if the product's market price falls below long-run average cost, the firm cannot survive.

[3]Despite the pronouncements of virtually every microeconomic theory textbook (an exception is Vandermeulen, *op. cit.*, "Linear Economic Theory," n. 7, p. 102), the firm's product supply curve is not identical to its marginal cost curve--or any part of its marginal cost curve--as (88) clearly shows. The conceptual experiment that generates a marginal cost curve is to parametrically vary output and record the changes in marginal cost, while the conceptual experiment that traces out a product supply curve is to parametrically vary the product's market price and observe the changes in the quantity of the product supplied by the firm.

[4]This second-order condition is

$$\partial^2\pi/\partial q^2 = -\partial^2 c/\partial q^2 < 0,$$

(continued on next page)

no external effects,[1] the firm's long-run output supply curve is positively sloped. Second (and more importantly as we shall now show), in the absence of external effects, the industry supply curve is merely the horizontal sum of the product supply curves of the firms comprising the industry and, hence, the industry supply must be positively sloped.[2]

Consider now a perfectly competitive industry composed of N firms. As discussed above, the traditional output supply curve of the firm is predicated on the *ceteris paribus* assumption that input prices are constant. It will be useful to make that assumption explicit by rewriting (86) and (88) for the j^{th} firm in the following way:

$$q_j = \theta_j(p|r_1,r_2 = \text{constant}) = \gamma_j(p|r_1,r_2 = \text{constant}), \quad (89)$$

where

$$\theta_j(p|r_1,r_2 = \text{constant}) = \kappa_j'^{-1}(p). \quad (90)$$

Given the assumption of no external effects, the industry supply curve may be obtained by adding the N individual supply functions in (89):

$$Q_S = \sum_{j=1}^{N} \gamma_j(p|r_1,r_2 = \text{constant}) = \Gamma(p|r_1,r_2,N = \text{constant}) \quad (91)$$

or

$$Q_S = \sum_{j=1}^{N} \theta_j(p|r_1,r_2 = \text{constant}) = \theta(p|r_1,r_2,N = \text{constant}). \quad (92)$$

(contd) which implies

$$\frac{\partial^2 c}{\partial q^2} = \frac{\partial(\partial c/\partial q)}{\partial q} > 0.$$

[1]These external effects arise when the firm's total cost depends not only on its own output level, but on the industry output level as well. See James M. Henderson and Richard E. Quandt, "Microeconomic Theory," 2d ed. (New York: McGraw-Hill Book Company, Inc., 1971), pp. 111-13.

[2]*Idem.*, p. 111.

Of course, the imposition of the expenditures requirement t may cause the number of firms in the industry to decline--as shown above,[1] $\partial N/\partial t < 0$. Thus, the aggregate supply function for the iron and steel industry can be expressed as

$$Q_S = S(p, r_1, r_2, t, N). \tag{93}$$

Let us turn now to a consideration of the demand side of the market for iron and steel castings. This demand is a derived one, since castings are produced inputs that are intermediate to final manufactured goods made by profit-maximizing firms--at least we shall assume that profit maximization is their objective. The market demand for castings can be obtained by aggregating the demand functions of the individual buyers, most of whom are private industrial firms.[2] The derivation of an individual firm's demand function for castings is directly analogous to the derivation of the input demand functions defined by (84). That is, the quantity of castings demanded by the firm depends on the market price of the firm's product, the market price of castings, and the market prices of other inputs that the firm uses. If we aggregate the demand functions of all buyers of castings, we obtain the corresponding market demand:

$$Q_D = D(\rho, p, r), \tag{94}$$

where ρ is an index of the market prices of the goods produced by purchasers of castings, p is the market price of castings, and r is the vector of market prices of other resources used by buyers of castings.

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[1]See *supra,* (79), p. B-15.

[2]Strictly speaking, some provision should be made in the model for public as well as private purchases of castings. However, less than five percent of the products of gray iron, malleable iron, and steel foundries were purchased by government agencies in 1972. Therefore, no attempt will be made here to include a theoretical construct for public "demand" for castings. See U.S. Department of Commerce, Bureau of the Census, *1972 Census of Manufactures, Industry Series: Ferrous and Nonferrous Foundries--SIC Industry Groups 332 and 336,* MC72(2)-33B (Washington, D.C.: GPO, 1974), pp. 33B3-33B4.